43 V and the phase shift can consequently be varied continuously between the two states shown in Figure 7 by applying a DC voltage between 0 and 42 V, which correspond to bridge heights of about 3 and 2 μm, respectively.

6. CONCLUSION

We have successfully designed and fabricated a CRLH-TL phase shifter that can be analogically controlled by means of a MEMS device. In addition, an accurate circuit model and a design strategy specific to the design of MEMS CRLH-TL have been presented and validated.

This work has shown that MEMS technology allows simultaneously adding variability, compactness, and monolithic integration to previously demonstrated capabilities of CRLH-TLs. It is therefore believed that MEMS technology, associated with appropriate modeling methods, should help widen the field of application of CRLH-TL metamaterials in the future.

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METAMATERIAL TRANSMISSION LINES WITH EXTREME IMPEDANCE VALUES

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ABSTRACT: In this work, it is demonstrated that it is possible to synthesize the metamaterial transmission lines with extreme characteristic impedances. The structures consist on left-handed microstrip lines based on the combination of complementary split rings resonators (CSRRs), which are etched in the ground plane, and series capacitive gaps. By means of an adequate design, it is possible to tailor the characteristic impedance of the lines over a wide margin. The realizable impedance values are not easily achievable through conventional transmission lines implemented in commercial microwave substrates. Several illustrative examples are provided and the limitations of the technique (relative to the maximum and minimum achievable impedances) are discussed. The compact dimensions of these artificial lines, related to the small electrical size of CSRRs, are also pointed out. These structures can be applied to the design of microwave components with severe requirements concerning characteristic impedance. © 2006 Wiley Periodicals, Inc.

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Key words: metamaterials; left-handed lines; complementary split rings resonators (CSRRs)

1. INTRODUCTION

Metamaterial transmission lines are microstructured propagating media with controllable electrical parameters and high dispersion. Among them, the so called left-handed (LH) lines, composed of electrically small unit cells, exhibit backward wave propagation. Two different approaches have been used so far for the synthesis of LH lines: (i) the composite right/left handed (CRLH) approach, where a host line is loaded with series capacitors and shunt inductors [1–4], and (ii) the resonant type approach, where electrically small resonant elements such as split rings resonators (SRRs) [5, 6], or their dual counterparts (complementary split rings resonators, CSRRs) [7, 8], are coupled to a host line. In both cases, there is a frequency band where wave propagation is backward. This behavior can be interpreted as due to the coexistence of negative effective permeability and permittivity in that band. For resonant type LH lines implemented by means of CSRRs, these resonant elements provide the negative effective permittivity [6], whereas the required negative permeability is achieved by etching series capacitive gaps in the conductor strip. These CSRR-based LH lines have been successfully applied to the design of compact microwave filters [9]. In this work, the main aim is to take benefit of the wide variation of the phase constant and characteristic impedance over the allowed band, to design artificial transmission.
2. THE BASIC CELL AND EQUIVALENT CIRCUIT

The basic cell for such lines is depicted in Figure 1 together with the lumped element equivalent circuit model. CSRRs are etched in the ground plane, underneath the positions of the series gaps. The gaps are modeled by the capacitance \( C_g \), whereas \( L \) models the inductance of the line. CSRRs are described by the resonators formed by the parallel combination of \( L_c \) and \( C_c \), and their coupling to the host line is modeled by the capacitance \( C \). To minimize dimensions as much as possible, these artificial transmission lines are preferably implemented by using a single cell. However, as will be later shown, this is not always possible. Nevertheless, coupling between adjacent CSRRs (in case of multiple stage devices) is ignored since it has been found that interaction between neighbor CSRRs is negligible for circular geometries [10]. Actually, in this circuit model, the line inductance \( L \) is ignored since we are only interested in the backward wave transmission band of the structure. However, to properly describe the transmission characteristics of the CSRR structure by ignoring \( L \), it is necessary that the resonator formed by \( L \) and the gap capacitance \( C_g \) exhibits a resonance frequency well above the intrinsic resonant frequency of the CSRRs, i.e.

\[
f_0 = \frac{1}{2\pi \sqrt{L_c C_c}}.
\]

The analysis of the circuit model of Figure 1(b) can be done with the help of the dispersion relation and Bloch impedance, which are given by [11]

\[
\cos \phi = 1 + \frac{Z_s(j\omega)}{Z_p(j\omega)},
\]

and

\[
Z_B = \sqrt{Z_s(j\omega)[Z_s(j\omega) + 2Z_p(j\omega)]},
\]

respectively, where \( \phi = \beta l \) is the phase of the basic cell, and \( Z_s \) and \( Z_p \) are the series and shunt impedances, respectively, of the basic circuit cell. In Figure 2 the variation of \( \phi \) and \( Z_B \) with frequency are depicted, within the LH allowed band, for a basic cell with the electrical parameters indicated in the caption. The structure is transparent (i.e. it supports propagating modes) within the interval delimited by the frequencies given by

\[
f_L = \frac{1}{2\pi} \sqrt{\frac{1}{L_c C_c + 4 \left( \frac{1}{C_g} + \frac{1}{C} \right)}},
\]

\[
f_H = \frac{1}{2\pi \sqrt{L_c C_c}}.
\]

Within this interval, \( \phi \) varies between 0 (at \( f_H \)) and \( -\pi \) (at \( f_L \)), whereas \( Z_B = 0 \) Ω at \( f_L \) and \( Z_B \to \infty \) at \( f_H \). This analysis and Figure 2 provide a good understanding of the transmission characteristics of the CSRR-loaded lines.
2 suggest that it is possible (by using these CSRR-based cells) to synthesize artificial transmission lines with impedances not easily achievable through conventional implementations. The main aim of this work is to demonstrate this, and to establish the limitations of this approach, which are mainly caused by the fact that $\phi$ and $Z_0$ are not fully independent parameters. In Figure 2 and the following figures, the phase $\phi$ is depicted in absolute value, but it is actually negative due to the LH nature of the structures.

3. THE SYNTHESIS METHOD

Let us call $\phi_c = \phi(f_c)$ and $Z_c = Z_0(f_c)$ the phase shift per cell and the desired line impedance, respectively, at the operating frequency $f_c$. From these input parameters, the parameters of the equivalent circuit model of the elemental cell can be inferred from the following equations [12]:

$$\begin{align*}
C_t &= \frac{1}{2\omega_0^2 Z_c} \frac{1 + \cos \phi_c}{1 - \cos \phi_c}, \quad (6) \\
L_c &= \frac{Z_c}{2} \left[ \frac{1 + \cos \phi_c, \omega_c (\omega_c^2 - \omega_0^2)(\omega_c^2 - \omega_0^2)}{\omega_c^2} \right], \quad (7) \\
C_s &= \frac{1}{L_c \omega_c^2}, \quad (8) \\
C &= \frac{2\omega_c^2 (\omega_c^2 - \omega_0^2) \sqrt{1 - \cos^2 \phi_c}}{Z_c \omega_c [\omega_c^2 (1 + \cos \phi_c)(\omega_c^2 - \omega_0^2) - 2\omega_c^2 (\omega_c^2 - \omega_0^2)]} \quad (9)
\end{align*}$$

where $\omega_c = 2\pi f_c$, $\omega_0 = 2\pi f_0$, and $\phi_c = 2\pi f_c$. These parameters must be positive and they should take reasonable values to lay out the basic cell with reasonable dimensions (i.e. within the tolerance limits dictated by the fabrication process). In Ref. 12, this aspect is considered, and it is concluded that there is a limited range of values of $f_c$ and $f_0$ that make the parameters given by expressions (6)–(9) to take positive values. Thus, there is an intrinsic limitation concerning the achievable bandwidth for such artificial lines. However, in practice, this is further limited by the dimensions of the resulting layout, which must be realizable. Since there is a link between impedance and phase (namely, as impedance varies between $Z_0 = 0$ at $f_c$ and $Z_0 \rightarrow \infty$ at $f_0$, the phase goes from $\phi = -\pi$ to $\phi = 0$), it is not possible to simultaneously obtain low line impedance and phase by means of a single cell, or, alternatively, a high impedance line with a phase shift close to $\phi = -\pi$. In the next section, these limitations will be discussed on the basis of artificial line design and implementation.

4. ILLUSTRATIVE EXAMPLES

Since many microwave circuits (admittance inverters, power dividers, filters, directional couplers, etc.) are based on 90° transmission lines or stubs, this has been the considered target value of $\phi$, whereas the operating frequency has been set to $f_c = 1.5$ GHz. The aim of this section is to study the maximum range of achievable line impedances with the previous phase constraint. We have used Eqs. (6)–(9), by giving tentative values to $\omega_0$ and $\omega_{01}$, to obtain the electrical parameters of the basic cell, and by following an optimization procedure, the layout of the structure has been generated. We have considered both the synthesis of high impedance (in reference to 50 $\Omega$) and low impedance artificial lines. Concerning high impedance lines, it has been found that it is possible to synthesize 150 $\Omega$ lines with 90° phase shift by means of a single cell. The layout and the simulated (using the Agilent Momentum commercial software) frequency response are shown in

Figure 3  Layout (a), insertion and return losses (b), and phase shift (c) for the artificial transmission line with characteristic impedance of 150 $\Omega$ and 90° phase shift at 1.5 GHz. Gap separation is 0.2 mm; CSRR dimensions are as follows: external radius $r_{ext} = 4.85$ mm, slot widths $c_{in} = 0.26$ mm, $c_{out} = 0.50$ mm, and slot separation $d = 0.14$ mm.
Figure 3. The impedance of the ports has been set to 150 $\Omega$ since, by doing this, total transmission at the target frequency must be obtained. The phase shift $\phi$ has been obtained from the simulated phase of the transmission coefficient, $\phi_{21}$, (with 50 $\Omega$ ports) following:

$$\cos \phi = \frac{\cos \phi_{21}}{T}, \quad (10)$$

where $T$ is the modulus of $S_{21}$. Inspection of Figure 3 indicates that $Z_B = 150 \Omega$ and $\phi = 90^\circ$ at 1.5 GHz. As compared to a conventional transmission line implemented on identical substrate (Rogers RO3010, dielectric constant $\varepsilon_r = 10.2$, thickness $h = 1.27$ mm) the length of the CSRR line is smaller (10.1 mm). The 90° conventional line is 22 mm long, and it is not implementable since it is only 6 $\mu$m wide, and this is not achievable with any known fabrication process in this type of substrates.

If the line impedance is set to 300 $\Omega$, which gives a width for the conventional line in the nanometre scale, it is not possible to implement a 90° line with a single cell. The reason is that as impedance increases, the phase decreases (as has been explained in earlier section). Therefore, to achieve the required phase shift, a solution in this case is to cascade two CSRR cells each contributing with 45° to the phase shift. The layout and simulated frequency response for this structure are depicted in Figure 4. Obviously, in this case, since two sections are needed, we do not take benefit of the small length of a single cell and the length of the artificial line (20.8 mm) is comparable to that of the conventional, but not realizable, structure (21.7 mm). This structure has been fabricated by cascading 50 $\Omega$ access lines. The photograph of the structure is shown in Figure 5. Also in this figure is depicted the frequency dependence of $Z_B$ and $\phi$ that has been inferred from measurement (by using the Agilent 8720ET VNA). The frequencies where $Z_B = 300 \Omega$ and $\phi = 90^\circ$ do not exactly coincide with the nominal values. This is due to fabrication related tolerances and to the fact that the impedance is very sensitive on frequency at these high values. Nevertheless, we are able to synthesize artificial lines with very high impedance values. Because of the presence of the access lines, the procedure to experimentally obtain the characteristic impedance and the phase shift corresponding to the CSRR structure (i.e. by excluding the access lines) is not as simple as just using Eq. (10) or representing the frequency response. We have obtained the characteristic impedance from the measured $S_{11}$ coefficient. This gives the impedance seen from the input port, and from the electrical length of the access lines, we can obtain the impedance seen from the interface between the access line and the CSRR cell. Once this impedance has been obtained, the characteristic impedance of the CSRR structure can be inferred from the well-known formula giving the input impedance of a loaded transmission line. To obtain the phase shift corresponding to the region of interest, we have experimentally obtained the phase of the transmission coefficient $\phi_{21}$ for the whole structure (including access lines). From it, we have inferred the phase shift according to Eq. (10), and by subtracting the experimentally measured phase shift of the access lines, we have finally obtained the phase shift $\phi$ of the CSRR structure.

With regard to the synthesis of low impedance lines, we have designed a single cell 25 $\Omega$ line with 90° phase shift. The layout and simulated frequency response are depicted in Figure 6. In this case, the conventional line is realizable, but the length of the artificial lines (8.1 mm) is substantially shorter (as compared with the length, 17.98 mm, of the conventional line).

We have also designed a single cell 90° and 10 $\Omega$ line at 1.5 GHz. The corresponding layout, frequency response, and phase shift are depicted in Figure 7. The conventional line is in this case 12.5 mm wide (i.e., implementable), whereas its length is 16.7 mm, i.e. longer than the artificial line (13.84 mm). This structure
has been also fabricated (see photograph in Fig. 8), and by following the procedure described before, we have experimentally obtained the dependence of $Z_B$ and $\phi$ on frequency (Fig. 8). Again there is some discrepancy between the frequencies where the measured electrical parameters of the line take the target values and the nominal frequency (1.5 GHz), which can be also attributed to certain inaccuracies in fabrication.

Essentially, in this work, the possibility of implementing artificial metamaterial transmission lines with impedance values in the range 10–300 $\Omega$ (with 90° electrical length) has been demonstrated. Theoretically, it is possible to achieve impedances beyond these limits, but in practice either the resulting topologies are not easily realizable or the sensitivity of impedance with frequency (high impedances) is too high (as is indeed the case for the 300 $\Omega$ line). This type of artificial transmission lines can find application

Figure 5  Photograph of the structure of Figure 4 with access lines (a), and dependence of the characteristic impedance and phase shift (b) with frequency [Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com]

Figure 6  Layout (a), insertion and return losses (b), and phase shift (c) for the artificial transmission line with characteristic impedance of 25 $\Omega$ and 90° phase shift at 1.5 GHz. Gap separation is 0.16 mm; CSRR dimensions are as follows: external radius $r_{\text{ext}} = 4.05$ mm, slot widths $c_{\text{in}} = 0.36$ mm, $c_{\text{out}} = 0.32$ mm, and slot separation $d = 0.18$ mm
in narrow band microwave circuits that require transmission lines and stubs with extreme impedance values. They are also interesting to reduce device size, as has been demonstrated from the comparison to conventional transmission lines. As has been seen in the preceding examples bandwidth is limited. It has been found that bandwidth is somehow wider for the small impedance structures. The reason is that in these structures, $C_g$ and $C$ are larger, and this enhances bandwidth [Eqs. (4)–(9)]. However, caution must be taken with Eqs. (4)–(9) since they are valid under the assumption that line inductance $L$ gives a negligible impedance as compared with the impedance associated to $C_g$. Moreover, strictly speaking, bandwidth in these structures is related to that frequency band where impedance lies within a certain range, and a more rigorous analysis, which is out of the scope of this paper, is required to study the effects of electrical parameters on bandwidth. Work is in progress to introduce additional elements to the topology of the basic cell to increase bandwidth.

5. CONCLUSIONS
In conclusion, it has been demonstrated that by loading microstrip lines with CSRRs and series gaps, it is possible to perform the synthesis of LH artificial transmission lines with extreme impedance values. Both high and low characteristic impedance lines have been designed and fabricated. While impedance and phase shift for such lines are not fully independent variables, the design of CSRR based lines with $90^\circ$ phase shift (useful in many applications) and different impedance values (as high as $300 \, \Omega$ and as

Figure 7 Layout (a), insertion and return losses (b), and phase shift (c) for the artificial transmission line with characteristic impedance of $10 \, \Omega$ and $90^\circ$ phase shift at 1.5 GHz. Gap separation is 0.18 mm; CSRR dimensions are as follows: external radius $r_{\text{ext}} = 4.06 \, \text{mm}$, slot widths $c_{\text{in}} = 0.51 \, \text{mm}$, $c_{\text{out}} = 0.62 \, \text{mm}$, and slot separation $d = 0.14 \, \text{mm}$

Figure 8 Photograph of the structure of Figure 7 with access lines (a), and dependence of the characteristic impedance and phase shift (b) with frequency [Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com]
low as 10 Ω) has been carried out. This method is useful to reduce the physical length of the transmission lines and to obtain impedance values not easily achievable through conventional printed lines.

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