Synthesis and applications of new left handed microstrip lines with complementary split-ring resonators etched on the signal strip

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Abstract: A new type of left handed microstrip lines implemented by means of complementary split-ring resonators (CSRRs) is proposed. The CSRRs are etched on the signal strip alternating with series gaps. Additionally, shunt connected stubs are introduced for the first time to the previous structure. The combination of these three elements, that is, the series gaps, the CSRR and the shunt inductance, enhances design flexibility. By this means, the ground plane is left unaltered and narrow band artificial transmission lines with good performance and compact dimensions can be synthesised. The lumped element equivalent circuit model of the structure is proposed and validated. To demonstrate the applicability of these new left handed transmission lines, two compact microwave components have been designed and fabricated: (i) a narrow band power divider and (ii) a band pass filter. The resulting power divider is 75% smaller than the conventional implementation and 50% smaller than previous power dividers implemented by means of CSRRs. The band pass filter performance is comparable to that of previous CSRR-based filters with ground plane etching, whereas its size is smaller. This work represents a significant progress on the design of microwave components based on CSRRs, that is, the approach is opened to those systems where the ground plane cannot be etched.

1 Introduction

Since the synthesis of the first left handed medium by Smith et al. [1] in 2000, different approaches have been developed to design artificial propagating structures supporting backward waves. In particular, in planar technology, Iyer and Eleftheriades [2], Oliner [3] and Caloz and Itoh [4] proposed almost simultaneously in 2002 the dual transmission line-based approach, where a host transmission line is periodically loaded with series capacitances and shunt inductances. Soon after the emersion of the previous approach, Martin et al. [5] succeeded in trying to apply Pendry’s split-ring resonators to synthesise a one-dimensional left handed medium by loading a coplanar waveguide (CPW) transmission line with split-ring resonators and shunt metallic strips. In a similar way, lefthanded propagation is also achievable using complementary split-ring resonators (CSRRs) [6]. These particles are the dual counterparts of the split-ring resonators and can be combined with capacitive gaps to periodically load a microstrip line and, thus, create a left-handed transmission line [7]. To guarantee CSRR excitation through an axial electric field (as is required), these elements have been previously etched on the ground plane, beneath the conductor strip of the host line [6, 7].

A new approach based on CSRRs has been recently proposed: the hybrid approach [8]. In these new structures, these resonators are combined with series capacitive gaps and shunt connected inductances to implement left-handed lines. The particularity of this approach is the presence of a transmission zero above the main left-handed transmission band [9]. The presence of this additional element (shunt inductance) and the transmission zero has led us to the implementation of compact filters with a good out-of-band
performance. Indeed, the hybrid approach can be considered an extension of the resonant-type approach, in which the design flexibility is enhanced because of the presence of the additional element.

During the last years, the resonant-type approach (as well as the hybrid approach) has revealed to be a good strategy to synthesise left handed transmission lines and microwave components with competitive performance and small dimensions [10, 11]. The study of the possibilities offered by this kind of left handed lines shows that, as in the case of the dual transmission line approach, resonant-type metamaterial transmission lines do also exhibit a composite right–left handed behaviour. Thus, broadband responses can be obtained when the balance condition is satisfied [12]. Using these artificial left handed transmission lines, it is also possible to control both the phase and the line impedance at any specific frequency and, thus, synthesise artificial lines with small dimensions, as well as extreme impedance values (sometimes not achievable through conventional lines [13]).

Although these structures have proved to be a good solution in many microwave applications, they can become unsuitable for those applications in which the bottom side of the substrate lies on the top of a metallic surface (holder), as in this case the resonators etched or printed on the bottom layer are disabled. As a solution to this problem, we have recently proposed a new type of left-handed lines based on CSRRs and implemented in microstrip technology [14]. In such artificial lines, all the constituent elements are etched on the top metal layer, so that the ground plane is left unaltered. The layout of a typical structure, which has been published in [14], is reproduced here for completeness (Fig. 1). The CSRRs are etched on the signal strip, alternating with the series gaps. The left-handed nature of the first transmission band has been demonstrated in [14]. However, line performance and bandwidth are limited mainly because of the moderate/small resonator to line coupling (which is in turn because of the decoupling between the series gaps and the CSRRs). To solve these limitations and gain flexibility, new left handed microstrip structures that combine CSRRs, series gaps and shunt stubs, all of them etched on the top metal level, are proposed in this work. The combination of all these elements facilitates the achievement of the required values of the different elements to obtain the desired response characteristics. To illustrate the possibilities of this new approach, two prototype devices have been designed and fabricated: a compact power divider and a band pass filter. In both designs, the ground plane is not etched. With these new left handed lines, many other applications can be envisaged. This new approach may open the door to new applications in which the limiting aspects of previous left handed microstrip lines based on CSRRs precluded their use.

2 New proposed left handed structures

As a first step to develop left handed CSRR-based structures with the absence of ground plane etching, the authors recently proposed a purely resonant left handed structure where CSRRs, etched on the signal strip, were combined with capacitive gaps [Fig. 1a] [14]. The frequency response of this structure is similar to those obtained in purely resonant left-handed structures with CSRRs etched on the ground plane [15]. However, the equivalent circuit model of the structure of Fig. 1b must include new elements to properly describe its behaviour. Specifically, since the series gaps and the CSRRs are decoupled, the fringing capacitances of the gaps, \( C_f \), must be added to the model, as Fig. 1b illustrates [14]. In the model reported in [15] to describe microstrip lines with CSRRs etched on the ground plane, this fringing capacitance contributes to the coupling capacitance between the line and the resonator, and hence it is included in \( C \). The most fundamental limitation of the left-handed lines based on the unit cell of Fig. 1 is bandwidth and, hence, losses. The presence of the series gaps alternating with the CSRRs decreases the electric coupling between the line and rings, and bandwidth is severely degraded.

In this work, losses and bandwidth are improved, modifying the previous structure. The new unit cell is based on the previous one, but includes a shunt inductive stub (Fig. 2) in order to enhance design flexibility and, thus, achieve the required characteristics (line impedance and phase) with broader bandwidths. The resulting structure is a hybrid cell, equivalent to that described in previous works [9], but with the advantage of presenting all its constituent elements on the top metallic layer of the substrate and, thus, avoiding ground plane etching. As expected for a hybrid structure, a transmission zero, because of the presence of the CSRRs, is located above the main left handed transmission band (Fig. 3), whereas such a
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complementary split-ring resonator is 6.62 mm and the distance between gap centers is 7.19 mm. For the hybrid cell, simulations is the Rogers R03010 with thickness \( h \) and dielectric constant \( \varepsilon_r \). The structure of Fig. 1, the hybrid cell can be designed to include the inductance which accounts for the presence of the grounded stub. In comparison with the series gaps and the CSRRs stubs alternate, we will assume that they are decoupled and, hence, we will consider the gaps and the rest of the structure separately. Thus, from the simulation of the isolated gaps, the values of \( C_g \) and \( C_f \) will be obtained. From the simulation of the structure formed by the rings and the shunt stub, we can obtain the rest of the parameters, in a similar way as it was done in [14]. The following expressions provide the normalised admittances corresponding to \( C_g \) and \( C_f \) from the simulated S-parameters of the series gap:

\[
Y_f = \frac{S_{11} + S_{21} - 1}{1 + S_{11} + S_{21}} \quad Y_g = \frac{S_{21}(Y_f + 1)}{1 + S_{11} - S_{21}}
\]

The five remaining parameters of the model can be extracted from the electromagnetic simulation of the structure without gaps. To do that, some relevant frequencies must be identified from the electromagnetic simulation, following a method analogous to the one fully described in [16]. One of these frequencies is the transmission-zero frequency, \( f_z \), which can be easily identified from the representation of \( S_{21} \), and is related to the electrical parameters of the circuit model through the expression

\[
\omega = 2\pi f_z = \frac{1}{\sqrt{L_c(C + C_f)}}
\]

On the other hand, the series and shunt impedances of this structure are

\[
Z_s = j\omega L_c \quad Z_f = j\omega L_d
\]

Regarding previous structures, the parameter extraction method of this cell had to include some variations that its transmission characteristics require. In fact, the shunt branch formed by \( L_d \), \( C_f \) and \( L_s \) exhibits not one, but two resonance frequencies, which can be inferred from the representation of the \( S_{11} \) parameter in the Smith Chart.

3 Parameter extraction

From the electromagnetic simulation (using the commercial electromagnetic software Agilent Momentum) of the hybrid unit cell, the parameters of the equivalent circuit model can be extracted. Through the later comparison of the electromagnetic and electrical responses, the circuit model of Fig. 2 can be validated. Because of the high number of involved parameters, we cannot infer them simultaneously from the simulation of the whole structure. However, since the series gaps and the CSRRs stubs alternate, we will assume that they are decoupled and, hence, we will consider the gaps and the rest of the structure separately. Thus, from the simulation of the isolated gaps, the values of \( C_g \) and \( C_f \) will be obtained. From the simulation of the structure formed by the rings and the shunt stub, we can obtain the rest of the parameters, in a similar way as it was done in [14]. The following expressions provide the normalised admittances corresponding to \( C_g \) and \( C_f \) from the simulated S-parameters of the series gap:

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Z_s = j\omega L_c \quad Z_f = j\omega L_d \quad \frac{1 - \omega^2 L_s(C + C_f)}{1 - \omega^2(L_cC + L_cC + L_dC) + \omega^2 L_sC L_dC}
\]

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Figure 2: Layout of the new left handed basic hybrid cell, including a shunt inductance (a), and its lumped element equivalent circuit model (b)

Figure 3: Comparison between the responses of a hybrid and a purely resonant cell

In both cases all the elements have been etched on the top metallic layer of the substrate. The addition of the shunt inductance provides a wider and more symmetric response, as well as lower losses. For the purely resonant cell, the width of the host line is \( W = 5.0 \) mm and gap separation is \( s = 0.17 \) mm. Complementary split-ring resonator dimensions are: width of the rings \( c = 0.46 \) mm, distance between inner and outer rings \( d = 0.26 \) mm and the area occupied by the complementary split-ring resonator is \( 6.62 \) mm \( \times 4.58 \) mm. The distance between gap centers is \( 7.19 \) mm. For the hybrid cell, \( W = 3.80 \) mm and \( s = 0.23 \) mm, whereas gaps total length, that is, the length in the orthogonal direction to the line is \( l = 4.97 \) mm. The complementary split-ring resonators have \( c = 0.16 \) mm, \( d = 0.15 \) mm, and a total area \( A = 3.36 \) mm \( \times 3.41 \) mm. Shunt stub length is \( l_s = 1.45 \) mm, and the via hole diameter is \( v = 0.3 \) mm. The distance between gap centers is \( 4.00 \) mm. The considered substrate in these simulations is the Rogers RO3010 with thickness \( h = 1.27 \) mm and dielectric constant \( \varepsilon_r = 10.2 \)

Moreover, the presence of a transmission zero above the left handed band makes possible the synthesis of a symmetric response. Although this is irrelevant, if these structures are intended to be used as artificial transmission lines, this aspect can be important for filter design, as will be seen in Section 4.
Analytically, these two frequencies can be obtained by forcing the denominator of $Z_\alpha$ in (4) to be zero. The solutions are given by

$$
\omega_{0 \pm} = \sqrt{\frac{\omega_0^2(1 + L_\alpha C_\alpha^2)}{2L_\alpha C_\alpha^2}} \pm \sqrt{\frac{\omega_0^2(1 + L_\alpha C_\alpha^2(2 + L_\alpha C_\alpha^2)) - 4L_\alpha C_\alpha^2 \omega_0^4}{2L_\alpha C_\alpha^2}}
$$

In (5), $\omega_0$ represents the transmission-zero angular frequency and $\omega_\alpha$ is the resonance frequency of the CSRR, which is given by

$$
\omega_\alpha = 2\pi f_\alpha = \frac{1}{\sqrt{l_\alpha C_\alpha}}
$$

When the shunt path to ground is opened, that is at $\omega_0+$ and $\omega_0-$, the reactive part of the input impedance seen from the ports is only because of the line inductance $L$, whereas the resistive part has the value of the opposite port impedance, $50 \, \Omega$. This results in an intersection between the curve of the $S_{11}$ and the unit normalised resistance circle in the Smith Chart. This allows us to calculate the value of the inductance $L$ from the reactive part of the input impedance at $f_0$, which can be obtained from the simulation. On the other hand, if we represent the phase shift $\beta l$, we can find the frequency at which $\beta l = 90^\circ$ where, according to

$$
\cos \beta l = 1 + \frac{Z_\alpha(j\omega)}{Z_p(j\omega)}
$$

the following condition is satisfied

$$
Z_\alpha(j\omega_0) = -Z_\alpha(j\omega_0)
$$

Thus, we have five conditions (given by expressions 2, 3, 5 and 8) and five parameters, so that circuit parameters are univocally determined. With this aim, the previous expressions have been inverted by using the commercial Maple 10 software. As a result of the application of these expressions have been inverted by using the commercial Maple 10 software. As a result of the application of this extraction method the structure of Fig. 2a, the values reported in Table 1 have been obtained.

The seven extracted parameters provide the electrical response represented in Fig. 4, which also includes the electromagnetic simulation of the whole structure. Only $C$ has been slightly tuned to adjust the electrical and electromagnetic responses. This can be understood taking into account that this capacitance can be faintly influenced by the presence of series gaps. As can be seen in Fig. 4, there is very good agreement between the electrical and the electromagnetic simulations and, thus, the model properly describes this hybrid left handed structure and is validated with these results.

<table>
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<th>$C_{\alpha}$, pF</th>
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**Table 1** Extracted element parameters for the structure shown in Fig. 2a

**Figure 4** Electromagnetic simulation of the structure depicted in Fig. 2 compared with the electrical response of the circuit model using the parameter values obtained through the extraction method described above (see Table 1)

For this structure, the dimensions are $W = 4.41 \, \text{mm}$ and $s = 0.16 \, \text{mm}$, whereas the length in the perpendicular direction to the line is $l = 7.02 \, \text{mm}$. The complementary split-ring resonators have $c = 0.16 \, \text{mm}$, $d = 0.15 \, \text{mm}$, and a total area $A = 4.01 \, \text{mm} \times 4.16 \, \text{mm}$. Shunt stub length is $l_s = 2.99 \, \text{mm}$, and the via hole diameter is $v = 0.22 \, \text{mm}$. The distance between gap centers is $4.75 \, \text{mm}$. The Rogers RO3010 substrate, with thickness $h = 1.27 \, \text{mm}$ and dielectric constant $\varepsilon_r = 10.2$, has been considered.

### 4 Design and applications of the new artificial left handed transmission lines

To demonstrate the possibility of applying this new kind of artificial transmission lines to the design of microwave devices, a band pass filter and a power divider have been designed and fabricated. The power divider has been implemented by means of two identical left handed transmission lines with characteristic impedance $Z_0 = 70.71 \, \Omega$ and electrical length $\beta l = -90^\circ$ (impedance inverters) at the desired operating frequency. The inverters have been designed to exhibit such electrical characteristics at 1.5 GHz. Fig. 5a shows the simulated insertion and return losses of the designed inverter that have been obtained through Agilent Momentum by using 70.71 \, \text{\Omega} ports. It can be seen that impedance matching is obtained at 1.5 GHz, showing that the line exhibits the desired characteristic impedance at the operating frequency. The dispersion characteristics, depicted in Fig. 5a, indicate that the required phase is achieved at the design frequency (actually, the magnitude of the $\beta l$ has been depicted in this figure).

This artificial line has been used for the design of a narrow band 1:2 power divider at 1.5 GHz. The device consists of two left-handed lines exhibiting the indicated characteristics and three 50\,\Omega access lines. A high miniaturisation level is achieved by implementing the 70.71 \, \text{\Omega} impedance inverters with left-handed lines. This can be appreciated in Fig. 6a,
where the layouts of the ‘metadivider’ and conventional divider are compared. The ‘metadivider’ exhibits an area, which is approximately four times smaller than the area of the conventional device. This size reduction is significantly higher than that achieved by implementing the power divider through left handed lines with CSRRs etched on the ground plane [10], where 50% size reduction was obtained (when compared with the conventional device).

The ‘metadivider’ of Fig. 6 has been fabricated on the Rogers RO3010 substrate with thickness $h = 1.27$ mm and dielectric constant $\varepsilon_r = 10.2$. The measured (by means of the Agilent 8720ET vector network analyser) frequency response of this device is depicted in Fig. 6(b) together with the electromagnetic simulations of the conventional and designed left-handed power divider. A very good agreement between the experimental and simulated curves for the ‘metadivider’ can be observed, except around the transmission zero, where the deviation is attributed to the fabrication process. This is because of the small dimensions of some of the geometrical parameters, which are close to the resolution limits. The designed device exhibits good performance around its operating frequency (1.5 GHz). Measured power splitting is in the vicinity of −3 dB (the slight degradation in $S_{21}$ is also attributed to fabrication related tolerances) and return losses are better than −20 dB. This power divider is, thus, suitable for narrow band applications. The main advantage of the device when compared with conventional implementations is the size demonstrated to be better than in other proposals based on left handed lines with CSRRs etched on the ground plane. Moreover, the ground plane has not been altered with the proposed implementation. Obviously, the bandwidth is limited, and hence this type of power dividers are of interest for narrow band applications where size reduction is the main concern. There are quite recent works on the field based on composite right–left handed transmission lines based on series capacitances and shunt inductances [18, 19] which have achieved an important miniaturisation level and wide bandwidths. The results in this work are not presented as an improvement with respect to the previously mentioned works, but as a demonstration of the application possibilities of these structures based on CSRR where the ground plane is left unaltered and there is no need of using lumped elements.

To further demonstrate the application possibilities of these hybrid left handed structures, a narrow band pass filter has also been designed and fabricated. In this case, the presence of the transmission zero above the transmission band becomes important to improve (when compared with the purely resonant approach) the upper out-of-band filter performance and to obtain a symmetric response. The fabricated device is a periodic structure formed by three identical left handed cells constituted by two series gaps, a shunt connected stub and a complementary spiral resonator (instead of a CSRR, see Fig. 7). In this design, the spiral resonator has been used in order to move the transmission zero further away from the
pass band of interest and to reduce unit cell dimensions [17]. This improves the out-of-band filter performance. The design methodology of this filter is that reported in [9], where a periodic band pass filter with CSRRs etched on the ground plane was designed and fabricated. As was done in [9], the part of the metallic film lying within the spirals has been removed to control the electric coupling (modelled by $C$). For the ease of comparison, the present band pass filter has been designed to exhibit an identical central frequency to that of the filter reported in [9], namely $f_0 = 1$ GHz, although the fractional bandwidth is smaller (4.5%). Following [9], each filter section has been designed to exhibit a phase shift of $\beta l = -90^\circ$ and a characteristic impedance of $Z_0 = 50$ $\Omega$ at the central filter frequency, and the topology of each cell has been optimised to achieve the required 3 dB bandwidth (i.e. that providing the nominal fractional bandwidth 4.5%). Fig. 7a shows the layout of the designed filter, whose dimensions are $12 \times 54$ mm$^2$ (dashed rectangle). These dimensions are significantly smaller than those of the periodic filter reported in [9] ($22 \times 45$ mm$^2$). The simulated (by excluding losses) and measured frequency responses of the filter are depicted in Fig. 7b. The slight discrepancies between full wave electromagnetic simulation and the experiment are because of fabrication related tolerances (i.e. several filter dimensions are too close to the limits of tolerance of the available drilling machine LPKF HF100) and to the fact that losses have been ignored in the simulation to verify that the filter has been properly designed. The measured fractional bandwidth of the fabricated filter is 5%. This narrow band explains the measured level of in-band losses, which is in the vicinity of 5 dB. The unloaded $Q$-factor of the resonators has been estimated to be $Q_u = 77$ from the standard expression given below [20]

$$\text{IL(dB)} = \frac{4.343 f_0}{\text{BW}} \sum_{i=1}^{N} g_i$$

(9)

In (9) IL represents the insertion losses, $g_i$ are the prototype element values and $\text{BW}/f_0$ is the equal ripple frequency bandwidth. Return losses are better than $-15$ dB in the interval 0.997–1.020 GHz and the measured out-of-band rejection is better than 60 dB up to 1.71 GHz. Finally, the filter exhibits a very symmetric frequency response. We would like to emphasise that the losses of the designed filter are because of the considered narrow band and the number of unit cells, rather than being related to an improper filter design.

5 Conclusion

In this work, a new type of one-dimensional left-handed structures based on CSRRs has been presented. All the elements of the structure lie on the top metal layer of the substrate, thus avoiding ground plane etching. In addition, shunt stubs have been introduced, in combination with the rest of the elements in the structure, to enhance design flexibility. This represents a clear advantage over previous CSRR-based left handed structures since small dimensions, good performance at the design frequency and device versatility (because of the fact that the structures can rest on a metallic holder) are simultaneously achieved. We would like to emphasise the fact that the structures presented in this work solve the problem of the left handed structures presented in [14], that is, the limited performance in terms of in-band losses. This limitation was attributed to the limited coupling between the line and the CSRRs. Introducing the shunt connected stubs, such a coupling is not relevant anymore and bandwidth and losses are improved.

An equivalent circuit model for these new left handed structures has been proposed and validated through a parameter extraction method. The new transmission lines are suitable for the design of compact narrow band microwave devices, as has been demonstrated through the design and fabrication of a power divider and a narrow band pass filter.

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7 References


