PARAMETRIC ANALYSIS OF MICROSTRIP LINES LOADED WITH COMPLEMENTARY SPLIT RING RESONATORS

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ABSTRACT: In this article, the effects of varying the geometry of complementary split ring resonators (CSRRs) in microstrip lines loaded with such resonant elements is analyzed. Specifically, from a parameter extraction technique previously proposed by some of the authors, the electrical parameters of CSRR-loaded lines (namely, the line inductance, the inductance and capacitance of the CSRR, and the coupling capacitance between the line and the CSRRs), are inferred. This analysis is of interest because, in spite of the existence of analytical models that predict the electrical parameters of these CSRR-loaded lines, the validity of these models is limited, and the parameter extraction method is necessary to accurately determine the electrical parameters from geometry. From the analysis carried out in this work, interesting conclusions for the design of CSRR-loaded lines are obtained. © 2008 Wiley Periodicals, Inc. Microwave Opt Technol Lett 50: 2093–2096, 2008; Published online in Wiley InterScience (www.interscience.wiley.com). DOI 10.1002/mop.23571

Key words: complementary split ring resonators (CSRRs); metamaterials; microstrip technology

1. INTRODUCTION

Complementary split ring resonators (CSRRs), first proposed by Falcone et al. [1], are useful particles for the synthesis of resonant-type metamaterial transmission lines. By periodically loading a host (microstrip) line with CSRRs (etched in the ground plane), a one-dimensional effective medium with negative permittivity in a narrow band below the resonance frequency of CSRRs is created [1]. By adding series capacitive gaps above the positions of the CSRRs, a negative effective permittivity and permeability simultaneously arise, and the structure exhibits left handed wave propagation [2]. The small size of the unit cell of such artificial lines and the possibility to tailor their dispersion diagram and characteristic impedance has lead to many microwave applications of CSRR-loaded lines, where device miniaturization [3] and/or performance improvement [4] have been key aspects. The field of filters deserves special attention, since it has been found that CSRRs are also appropriate resonators either for the synthesis of novel compact planar filters [5–8] or to improve the performance (through spurious pass band suppression) of existing ones [9].

A typical unit cell of a microstrip line loaded with CSRRs and its lumped element equivalent T-circuit model are both depicted in Figure 1. In this model, which has been reported before [10], $L$ is the line inductance, $C$ is the coupling capacitance between the line and the CSRRs, and $C_c$ and $L_c$ model the reactive elements of the CSRRs, which are described by parallel resonant tanks. It has been demonstrated that this circuit model accurately describes the behavior of the unit cell structure and also the behavior of multiple cell structures, where the coupling between adjacent CSRRs is negligible [11, 12]. This has been confirmed from a parameter extraction method reported in Ref. 11. In this work, this method is applied to extract the parameters of a single-cell CSRR loaded line where the geometrical parameters of the CSRRs are varied. This includes the CSRR radius, as well as the width and separation of the slots (see Fig. 1 for details). From this parametric analysis, design guidelines for the CSRR-loaded lines will be obtained. This is useful since, in spite of the fact that there are analytical models


that link the geometrical and electrical parameters of CSRRs [10], these models are valid under restrictive conditions, and they can only be used to provide an initial seed for the CSRR geometry, which needs optimization.

2. PARAMETRIC ANALYSIS

For the study, the parameters of the Rogers RO3010 substrate with dielectric thickness $h = 1.27$ mm are considered (the dielectric constant is $\varepsilon_r = 10.2$). Losses are excluded in the simulations, hence metal thickness and $\tan \delta$ are irrelevant for us. This “lossless” substrate is considered since the presence of losses may cause certain “noise” in the determination of the electrical parameters of the CSRRs. Nevertheless, the parameter extraction method perfectly works under lossy conditions since it has been applied to determine the electrical parameters of fabricated structures and these parameters have been inferred from the measured reflection and transmission coefficients (see Ref. 11 for details). With regard to the host line, it has a characteristic impedance of $Z_0 = 50 \, \Omega$, which corresponds to a line width of $W = 1.16$ mm in such substrate.

In the first analysis, we set $c = d = 0.2$ mm and we obtain the electrical parameters of the unit cell model as a function of the average radius $r_0$ of the CSRR. The capacitance of the CSRR, $C_c$, is the edge capacitance of a metallic disk of radius $r_0 = c/2$, surrounded by a metal (ground plane) at a distance $c$ of its edge. The CSRR inductance, $L_c$, is given by the parallel connection of the two inductances connecting the inner disk to the ground. Each inductance is given by $L_0/2$, where $L_0 = 2\pi r_0^2$ and $L_{pul}$ is the per unit length inductance of the CPWs connecting the inner disk to the ground [10]. According to this, one expects a linear dependence of $C_c$ and $L_c$ with $r_0$, or, in other words, we expect no dependence of these parameters with $r_0$ when they are divided by the average perimeter of the CSRR, namely, $2\pi r_0$. Such per unit length parameters, $C_{pul}$ and $L_{pul}$, have been inferred from the parameter extraction method for different values of the average radius, ranging in the interval $0.7 \, \text{mm} < r_0 < 8 \, \text{mm}$. The results are depicted in Figure 2. The parameters that result when dividing $L$ and $C$ by the average CSRR perimeter ($L_{pul}$ and $C_{pul}$), even though they do not have a physical meaning, have also been depicted in Figure 2. $L_{pul}$ and $C_{pul}$ are constant (except for small values of $r_0$, where these values become unrealistic), this being an expected result. However, $C_{pul}$ and $L_{pul}$ exhibit a certain variation with $r_0$ for small $r_0$ values, and then they saturate. This variation of $C_{pul}$ and $L_{pul}$ not accounted for by the analytical model [10], is attributed to the presence of the strip line at the other side of the substrate, the influence being more significant in small CSRRs.

The second analysis that has been carried out is the effect of varying $c$ and $d$, for a fixed value of the average radius. This has been set to $r_0 = 8$ mm, that is, in the region of saturation of the precedent analysis. Specifically, we have considered $c = d$ in this analysis. Obviously, this is not necessary, but by making this assumption, we reduce the number of variables. The effects of varying $c$ and $d$ on $L_c$ and $C_c$ are depicted in Figure 3. $L_c$ is weakly dependent on $c$ and $d$, whereas $C_c$ varies significantly. The ratio between these parameters is depicted in Figure 4. In the considered range ($0.05 \, \text{mm} < c = d < 0.75 \, \text{mm}$), the ratio $L_c/C_c$ changes roughly between $900\,\text{H/F}$ and $1600\,\text{H/F}$. Thus, this result can be obtained for design purposes. Namely, from the required values of $L_c$ and $C_c$ (determined from circuit specifications), the ratio $L_c/C_c$ can be obtained and from the curve of Figure 4, $c$ and $d$ can be determined. Once determined $c$ and $d$, the average radius can be modified to obtain the required value of either element. Obviously, the achievable values are limited since the CSRR can be neither large nor small.

It is worth mentioning that this study reveals that by using CSRRs, the ratio between the reactive elements ($L_c/C_c$) is limited to the resulting interval (see Fig. 4). If different values are necessary, other resonators such as the complementary spiral resonator

![Figure 2](image-url)  
*Figure 2* Variation of $C_{pul}$, $L_{pul}$, $L_{pul}$, and $C_{pul}$ with the average radius, $r_0$, of the CSRRs

![Figure 4](image-url)  
*Figure 4* Dependence of the ratio $L_c/C_c$ with $c = d
(CSR) can be used (the circuit model of CSRs and its relationship with the circuit model of CSRRs has been reported before [10]).

3. THE PRESENCE OF A SERIES GAP IN THE STRUCTURE: DISCUSSION

In the preceding analysis, the presence of a series gap in the structure has been ignored. Indeed, the parameter extraction technique has also been applied to microstrip lines loaded with both CSRRs and series capacitive gaps etched above the position of the CSRRs [11]. With the presence of the series gaps, the structure exhibits a pass band behavior with left handed wave propagation in the first allowed band, as has been exhaustively reported [2, 10]. It has been previously found that the extracted parameters of CSRRs ($L_c$ and $C_s$) do not vary with and without the presence of the series capacitive gaps [11]. However, it has been found that the coupling capacitance $C$ is significantly enhanced when the series gap is present. The series capacitive gap introduces an extra capacitance to ground (fringing capacitance), but this can not actually explain such enhancement.

Actually, the series gap must be modeled by means of a π-circuit composed of three capacitors in either branch ($C_s$ for the series branch and $C_f$ for the fringing/parallel capacitance). According to this, the circuit model of the unit cell structure with the presence of the series gaps, should be modeled as Figure 5(a) illustrates, where $C_L$ is the contribution of the line capacitance. Obviously, from π–T transformation, the circuit model of Figure 5(b), which is the reported model of microstrip lines loaded with CSRRs and series gaps, is obtained, but the values of $C_{par}$ and $C$ do not actually have a physical interpretation. Indeed, $C_g$ and $C$ can be expressed in terms of $C_s$ and $C_{par} = C_f + C_L$, according to:

$$C_g = 2C_s + C_{par} \quad (1)$$

Thus, if $C_s$ is small, the coupling capacitance can be very large, and this may occur in many practical situations. The main conclusion of this section is that the model of CSRR and gap-loaded lines reported previously is correct. It provides a good description of the structures, but it actually results from a transformation of another circuit model [Fig. 5(a)] where all the parameters have a physical interpretation.

4. CONCLUSIONS

In conclusion, from a previously reported parameter extraction method, we have extracted the parameters of the circuit model of a CSRR-loaded line with different CSRR geometries. The results obtained can be of interest to aid the design of planar microwave components based on CSRRs, since the analytical models of CSRR are valid under restrictive conditions. We have also discussed the effects of a gap capacitance etched in the line, and we have modified the equivalent circuit model to account for the enhanced coupling between the line and the CSRRs when the gap is present. Actually, the model can be transformed to the previous reported model with modified parameters. From this, the enhancement of the coupling capacitance when the gap is present can be perfectly explained.

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$$C = \frac{C_{par}(2C_s + C_{par})}{C_s} \quad (2)$$


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MEMS-RECONFIGURABLE BANDPASS FILTER

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ABSTRACT: This article presents a MEMS-based bandpass filter with a reconfigurable frequency response. The proposed device consists of a stub-loaded Branch-Line Coupler in artificial transmission line technology, resulting in a very compact structure. Furthermore, by using a micro electro mechanical system resistive switch, a reconfigurable frequency response has been obtained. © 2008 Wiley Periodicals, Inc. Microwave Opt Technol Lett 50: 2096 –2099, 2008; Published online in Wiley InterScience (www.interscience.wiley.com). DOI 10.1002/mop. 23570

Key words: micro electro mechanical system; artificial transmission lines; reconfigurability; branch-line coupler; bandpass filter

1. INTRODUCTION

High performance bandpass filters are essential components in communication systems, and in the last years several design techniques have been developed to achieve both low insertion loss and high selectivity filtering responses [1–4]. Among these, particularly attractive is the solution proposed in [4], where a branch-line directional coupler (BLC) has been employed to achieve a high-selectivity filter based on signal-interference.

Furthermore, in many modern wireless communication systems there has been considerable interest in developing miniaturized devices with a reconfigurable frequency response [5–9]. Radio frequency (RF) microelectromechanical systems (MEMS) are one of the most promising technologies to meet these requirements providing low insertion loss, low-power consumption, and excellent linearity [7–10].

The bandpass filter proposed in this letter is based on the approach suggested in [4]; specifically, the stub-loaded BLC has been designed in artificial transmission lines (ATLs) technology leading to a very compact device; furthermore, to obtain a variable length loading stub, a micro electro mechanical system (MEMS) series resistive switch has been employed. This way, the filter selectivity and its stopband rejection can be customized acting on the switch state (ON/OFF).

The article is structured as follows: in Section 2 the fundamentals of the ATLs technology are given, later on, Section 3 illustrates the proposed reconfigurable bandpass filter. Finally results are given in Section 4 and some conclusions drawn in Section 5.

2. ARTIFICIAL TRANSMISSION LINES

The artificial transmission line concept is based on the possibility to model any dielectric with a distributed L–C network in low/high-pass topology. Relating the per-unit-length capacitance (C0) and inductance (L0) to the electric permittivity (\(\varepsilon\)) and magnetic permeability (\(\mu\)) of the medium as follows:

\[ |\mu| = \frac{Z_{\text{in}}}{j\omega} = \frac{j\omega L_0}{j\omega} = L_0, |\varepsilon| = \frac{Y_{\text{in}}}{j\omega} = \frac{j\omega C_0}{j\omega} = C_0 \]  \(1\)

The transmission line (TL) propagation constant and its characteristic impedance become:

\[ \beta_0 = \sqrt{-jZ_{\text{in}}} = \omega \sqrt{|\varepsilon||\mu|} Z_0 \quad Z_0 = \frac{Y_0}{C_0} = \sqrt{|\varepsilon|} \]

Consequently, the TL phase velocity is given by:

\[ \frac{1}{v_{p,TL}} = \frac{1}{\sqrt{L_0 C_0}} \]  \(3\)

Now, let us consider the periodic structure reported in Figure 1 with a unit cell made of a TL loaded with a shunt capacitance; from the effective medium theory, one can derive that at frequencies corresponding to wavelengths much larger than the TL-length (d), the artificial transmission line (ATL) behaves as an “effectively homogeneous” TL, whose effective characteristic impedance and phase velocity are:

\[ Z_{\text{eff,TL}} = \sqrt{(C_0 + C_0/d)} \]

\[ \beta_{\text{eff,TL}} = \frac{1}{\sqrt{L_0 (C_0 + C/d)}} \phi_{\text{eff,TL}} = \frac{Nd_0}{v_{\text{eff,TL}}} \]

\[ = Nd_0 \left[ L_0 (C_0 + C_0/d) \right]^{1/2} \]  \(4\)

Being \(C_0\) the loading element, whilst \(N\) is the number of unit cells.

From (3) and (4), it is evident that the ATL is characterized by a lower effective characteristic impedance and phase velocity. Consequently, the same electrical length can be achieved with a

![Figure 1](image-url) Unit cell of an artificial TL obtained by periodically loading a conventional TL with a shunt capacitance and corresponding realization in microstrip technology.