Composite right-/left-handed coplanar waveguides loaded with split ring resonators and their application to high-pass filters

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Abstract: In this study, it is demonstrated that coplanar waveguides periodically loaded with split ring resonators (SRRs) and shunt connected elements can be applied to the synthesis of composite right-/left-handed transmission lines with very good high-pass filter characteristics. By balancing the line and forcing the characteristic impedance around the transition frequency to be in the vicinity of 50 Ω, a broad transmission band with low insertion losses is obtained. Owing to the presence of the transmission zero to the left of the left-handed band, severe cut-off is also obtained. A prototype high-pass filter device has been designed, fabricated and characterised to illustrate the possibilities of the approach.

1 Introduction

Composite right-/left-handed (CRLH) lines are artificial lines that exhibit backward (or left-handed) wave propagation at low frequencies and forward (or right-handed) wave propagation at high frequencies. Typically, a frequency gap is present between the left-handed and the right-handed bands of the line, unless it is designed to be balanced. In this case, the gap collapses and there is a continuous transition between the left-handed and the right-handed band at the so-called transition frequency, where the phase velocity is ideally infinity (with a finite group velocity).

The first lines exhibiting CRLH behaviour were designed by loading a host line with series capacitances and shunt inductances [1–4] (CL-loaded approach). However, artificial lines inspired on the first left-handed medium reported by Smith in 2000 [5] were also reported in 2003 by some of the authors [6]. Such lines consist on a host coplanar waveguide (CPW) periodic loaded with split ring resonators (SRRs) [7] and shunt connected strips (acting as inductive elements). In [6], left-handed wave propagation was pursued. It was demonstrated and interpreted as because of the negative effective permeability and permittivity provided by the SRRs and shunt strips, respectively. The structure reported in [6] exhibits a narrow left-handed band and the interest was to demonstrate the possibility of implementing left-handed structures in planar technology by means of SRRs. Thus, the characterisation at higher frequencies was not considered, and the CRLH behaviour of SRR-loaded lines was not pointed out. Indeed, the left-handed behaviour of the structures reported in [6] was also explained from the equivalent circuit model, also presented in [6] and improved in a recent paper by some of the authors [8].

In 2004, the complementary split ring resonator (CSRR) was presented for the first time [9], and applied to the synthesis of left-handed lines in microstrip technology [10]. In this case, a host microstrip line was loaded with CSRRs and series capacitive gaps. The negative effective permeability and permittivity was provided by the series gaps and CSRRs, respectively. Artificial lines based on CSRRs have been applied to many devices such as filters [11–18], diplexers [19], couplers [20] etc., but it was not until 2007, that the CRLH behaviour of such lines based on CSRRs was reported [21]. It was shown in [21] that by balancing these lines, a very broad transmission band can be achieved, where one portion of the band corresponds to backward wave transmission and the other to forward wave
transmission. These CSRR-based artificial lines have been subsequently applied to the design of band pass filters for ultra wide band (UWB) applications [17, 22].

The purpose of this paper is to demonstrate that balanced CRLH transmission lines based on SRRs can also be applied to the design of broadband filters in CPW technology. Indeed, the CRLH behaviour of these lines was first mentioned in [23], and experimentally demonstrated in [24]. In the present paper, the objective is to design a high-pass filter and to provide design guidelines to obtain good filter characteristics. Some considerations concerning the equivalent circuit model of the structure, different than that reported in [24], will be also included.

2 Topology and circuit model of the CRLH CPW transmission lines based on SRRs

The typical topology (unit cell) of the CRLH CPW structures based on SRRs is depicted in Fig. 1. It consists of a host CPW loaded with a pair of SRRs on the back substrate side and a pair of shunt connected inductive elements, implemented by means of a strip or a meander. Actually, the meander topology is not necessary, but it may be convenient in some applications requiring large values of the inductance. As compared to the structure reported in [24], we do not make use of wire bridges. The first equivalent circuit model of these structures was reported in 2003 [6] by some of the authors. Such model is reproduced here for completeness (Fig. 2a). According to this model, \( L_s \) and \( C_s \) are the inductance and capacitance of the SRR, the line inductance and capacitance are \( L \) and \( C \), respectively, the magnetic coupling between the line and the SRR is modelled through the mutual inductance \( M \), and, finally, the shunt inductive elements are described by means of the inductance \( L_p \). This circuit can be easily transformed into that depicted in Fig. 2b, where the new elements \( L_s' \) and \( C_s' \) are related to the \( L_s, C_s \) and \( L \) through the following equations

\[
L_s' = M^2 C_s \omega_0^2 \quad (1)
\]

\[
C_s' = \frac{L_s}{M^2 \omega_0^2} \quad (2)
\]

\[
L' = 2L - L_s' \quad (3)
\]

The circuit model of Fig. 2b is that considered in [24], but the authors of [24] claim that \( L_s' \) and \( C_s' \) are the inductance and capacitance of the SRRs, and this assumption is not valid [although \( L_s' \) and \( C_s' \) are related to the SRR elements through (1) and (2)]. However, the most important aspect concerning the circuits of Figs. 2a and 2b is that, according to
this model, the transmission zero of these structures, given by
the resonance frequency of the SRRs, does not depend on $L_p$.
If this is true, then we do not expect changes in the
transmission zero by varying the characteristics of the
shunted strips or meander. However, it has been
demonstrated that the transmission zero frequency is
sensitive to the characteristics of the shunt element. In [8],
it was demonstrated that rather than the model of Fig. 2a, it is
the circuit model depicted in Fig. 3a that properly describes
the structure. This new model is more realistic since the
shunt inductance is placed between the two inductances that
model the line and are magnetically coupled to the SRRs.
The model of Fig. 3a can also be transformed to the
circuit of Fig. 3b. The element values are related to those
of the original model (Fig. 3a) through the following
equations [8]

$$
L' = 2M^2 C_s \omega_0^2 \left[ 1 + \frac{(L/4L_p)}{1 + (M^2/2L_pL_s)} \right]^2 (4)
$$

$$
C'_s = \frac{L_s}{2M^2 \omega_0^2} \left[ 1 + \frac{M^2/(2L_pL_s)}{1 + (M^2/4L_p)} \right]^2 (5)
$$

$$
L' = \left( 2 + \frac{L}{2L_p} \right) \frac{L}{2} - L'_s (6)
$$

$$
L'_p = 2L_p + \frac{L}{2} (7)
$$

![Figure 3 Improved circuit model](image)

*a For the basic cell of the left-handed CPW structure
*b Transformed model from a
In this case, the magnetic wall concept has not been used
where $\omega_0^2 = 1/L_sC_s = (2\pi f_0)^2$, and the analysis of these
equations reveals that the transmission zero frequency, given by

$$
\omega_z = \frac{1}{\sqrt{L_sC_s[1 + (M^2/2L_pL_s)]}} (8)
$$
does depend on $L_p$, in agreement with experimental data (not
shown here but reported in [8], and corroborated from
electromagnetic simulation in [25]). Thus, in summary, the
model of Fig. 2b is formally correct, but the parameter
interpretation given in [6, 24] was not correct. The elements
of the model of Fig. 3b are related to the physical parameters
of the structure through the transformation given in (4)–(7).
We have recently published a paper where a parameter
extraction method for the elements of the model of Fig. 3b
is reported, and very good agreement with experimental data is
obtained [26]. This supports the validity of the proposed
model of Fig. 3a, which can be transformed to that of Fig. 3b
through (4)–(7).

### 3 High-pass filters based on balanced CRLH SRR-based lines

As pointed out in previous works [21, 23], balanced CRLH
lines based on the resonant-type approach are very interesting
for the design of high-pass filters with a sharp cutoff. For the
design of this kind of filters, it is necessary to balance the
lines, that is, to force the shunt resonance and the series
resonance (namely, that frequency that nulls the impedance
of the series branch) of the $\pi$-circuit model of Fig. 3a to be
identical. From this, the following condition is obtained

$$
L'_pC_s = \frac{2L'_sC'_s}{L'_s + L'_s} (9)
$$

and by using (4)–(7), it follows that the element values of the
circuit of Fig. 3a must satisfy

$$
(4L_p + L) \frac{C_s}{2} = \frac{C_s(2LL_pL + LM^2 - 4M^2C_s\omega_0^2L_pL_s - M^2C_s\omega_0^2L_sL_p)}{L_pL} (10)
$$

In order to obtain low insertion losses in the transmission
band, it is necessary that the characteristic impedance of the
structure is set close to the reference impedance of
the ports (normally 50 $\Omega$). For a $\pi$-circuit model, the
characteristic impedance is given by

$$
Z_B = \frac{Z_s^2Z_p}{\sqrt{Z_p + Z_s}} (11)
$$

where $Z_s$ and $Z_p$ are the series and shunt impedances, respectively, of this model. By using the
element values of the circuit of Fig. 3b, expression (11) gives
(see (12))
\[
\begin{align*}
\omega_s^2 &= \frac{2}{L_0 C} (2\pi f_s)^2, \\
\omega_0^2 &= \frac{(L' + L_0')(C'_0 L_0')}{(2\pi f_0')^2}
\end{align*}
\]
and this expression is simplified to
\[
Z_B = \left[ \frac{-\omega_s^2 L_0' (L' + L'_0)}{\omega_0^2 (L' + L'_0)(1 - (\omega^2/\omega_0^2))}\right]^{1/2}
\] (13)

for balanced lines. The analysis of expression (12) indicates that at the upper limit of the left-handed band, the impedance tends to zero or infinity, depending on the relative value of the series and shunt resonance. The same occurs at the lower limit of the right-handed band. However, if the resonances are identical (balanced case), the characteristic impedance exhibits a continuous variation in the vicinity of the transition frequency, and the characteristic impedance is roughly constant in a very wide band. The three considered cases are depicted in Fig. 4. Thanks to the fact that the characteristic impedance experiences small changes in a wide band for a balanced structure, it is possible to implement wide band filters by using a periodic structure of balanced unit cells. The unit cell can be properly designed to exhibit 50 \( \Omega \) characteristic impedance, which is the reference impedance of the ports, over a wide frequency range within the passband. This provides good matching and good insertion and return loss levels, as can be observed in its frequency response (Fig. 6).

The transmission zero frequency can also be used to force another condition for the element values of the circuit of Fig. 3b. Since typically these structures exhibit a very sharp cutoff, the transmission zero must be located at that frequency where a certain level of rejection is required. Obviously, by increasing the number of unit cells, rejection in the stop band increases.

\[
Z_B = \left[ \frac{-\omega_s^2 L_0' (L' + L'_0)(1 - (\omega^2/\omega_0^2))}{(1 - (\omega^2/\omega_0^2))} \right]^{1/2}
\] (12)

Figure 5 Photograph of the fabricated filter, dimensions are: external radius of the SRRs \( r_{ext} = 4.68 \text{ mm} \), width of the rings \( c = 0.223 \text{ mm} \) and rings separation \( d = 0.237 \text{ mm} \)

The shunt inductance has been implemented by means of a meander with width \( w_m = 0.2 \text{ mm} \) and separation distance \( z_m = 0.2 \text{ mm} \). Furthermore, the gap distance between the line and the ground planes is \( g = 0.16 \text{ mm} \).
On the basis of these considerations, we have designed an order-3 high-pass filter with the transmission zero located around $f_z = 1.4$ GHz. The layout of the device is depicted in Fig. 5 (dimensions of the dashed area are as small as $20.8$ mm $\times 35.2$ mm). The device has been fabricated on a substrate with dielectric constant $\varepsilon_r = 11.2$ and thickness $h = 1.27$ mm (the photograph of the fabricated filter is also depicted in Fig. 5). The simulated (by means of the Agilent Momentum commercial software [27]) and measured frequency response of the device are both depicted in Fig. 6. Good agreement between simulation and experiment results (except some frequency shift attributed to fabrication-related tolerances). The measured rejection in the stop band is better than $30$ dB, and insertion losses in the transmission band are good ($IL = 2$ dB between 1.40 and 2.48 GHz). At high frequencies, the model is not valid since the lumped element approximation fails, and transmission is degraded. Nevertheless, these filters exhibit a very broad band with a sharp cutoff and controllable rejection in the stop band. To gain further insight on the characteristics of the designed structure, we depict in Fig. 7 its dispersion diagram inferred from electromagnetic simulation, experiment and from the circuit of Fig. 3b with the extracted parameters. Although perfect balance is not achieved, the effects of the small gap on the transmission characteristics of the filter are irrelevant, as can be seen in Fig. 6.

4 Conclusions

In conclusion, it has been demonstrated in this paper that CRLH lines implemented by loading a host CPW with SRRs and shunt connected elements (meandered strips) can be applied to the design of high-pass filters with sharp cutoff. To this end, the artificial lines must be balanced and the characteristic impedance must be forced to vary smoothly in the vicinity of the reference impedance of the ports (50 $\Omega$). From the improved circuit model of the structures, we have provided design equations. Such design equations determine conditions for the element values of the transformed $\pi$-circuit model, which are related to the element values describing the artificial lines (and hence configuring the direct model of the structure) through specific transformations. A prototype device high-pass filter has been designed, fabricated and characterised. The dimensions of manufactured device are small and its performances have found to be in good agreement with those predicted by theory and simulation.

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6 References


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