Miniaturised and harmonic-suppressed rat-race couplers based on slow-wave transmission lines

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Abstract: In this study, a compact rat-race hybrid coupler with harmonic suppression based on slow-wave transmission lines (SW-TLs) is presented. Such artificial lines are implemented by periodic loading a host microstrip line with series meandered inductors and shunt patch capacitors. The presence of both loading elements has a twofold effect, i.e. phase velocity reduction (due to the Bragg effect, inherent to periodicity), and the generation of a controllable stopband in the frequency response (due to the Bragg effect). It is shown that by designing the unit cell of the periodic line with an electrical length of 45°, at least the first five harmonic bands of the rat-race coupler are efficiently suppressed, keeping the band of interest unaltered. Moreover, 79% size reduction, as compared to the ordinary coupler, is achieved in the reported SW-TL-based prototype.

1 Introduction

Size reduction and spurious/harmonic suppression have been (and are still) two challenges in the design of distributed microwave components. Miniaturisation is of special interest in device topologies involving multiple transmission line sections, such as high-order filters or couplers, among others. Spurious and harmonic bands are inherent to distributed components, and may be the cause of interferences or undesired signals that may degrade system functionality. Therefore, the suppression of such bands is a due in certain applications, and achieving that purpose without the penalty of increasing circuit size is of the highest interest.

Slow-wave transmission lines (SW-TLs) based on periodic structures are good candidates to simultaneously reduce circuit size and reject spurious/harmonic bands [1–3]. In such artificial lines, the phase velocity is smaller than the one of their ordinary counterparts implemented on the same substrate. Therefore, signal wavelengths are reduced, with direct impact on circuit compactness. Moreover, due to the Bragg effect, related to periodicity, periodic SW-TLs exhibit stopbands at controllable frequencies, useful for spurious/harmonic suppression.

Most periodic SW-TLs are either based on capacitive [4–15] or inductive [16–19] loading, and few of them are implemented by combining both reactive elements [20–23] (other SW-TLs loaded with distributed elements or combinations of distributed/semi-lumped components have been also reported [24–30]). The advantage of simultaneous inductive and capacitive (LC) loading is major design flexibility and the possibility to achieve further levels of miniaturisation, as compared to SW-TLs based on a single reactive element. In [23], a compact and harmonic suppressed branch-line coupler based on LC-loaded SW-TLs was reported. There are many papers in the recent literature devoted to the miniaturisation and harmonic suppression of branch line couplers [9, 12, 24–39]. Among them, the devices presented in [31, 33, 35, 38] use either capacitive or both capacitive and inductive loading, whereas the papers [25, 27] present branch-line couplers with size reduction based on T-shaped structures. However, fewer efforts have been dedicated to achieve that objective in rat-race couplers [9, 29, 40–48]. In [24, 26, 29, 45], miniaturisation in the considered couplers is achieved by means of open-stub loaded lines. In [43, 47], the authors use optimisation techniques (based on space mapping) for the implementation of harmonic suppressed rat-race couplers based on semi-lumped elements. In the works [44, 48], the authors use fractal and T-shaped structures, respectively. Finally, in [40–42, 46], the capacitive or capacitive/inductive loaded lines are used to reduce the size of the proposed couplers. In this paper, based on the periodic LC-loaded SW-TLs first proposed in [49], we report a miniature rat-race hybrid coupler able to suppress at least up to the fifth harmonic band. As compared to previous works devoted to size reduction and harmonic suppression of rat-race couplers, in this paper we report a design methodology based on the Floquet analysis of periodic structures, and we justify the convenience to use unit cells with a specific electrical length (to be discussed later) in order to efficiently suppress the harmonic bands leaving the band of interest unaltered.

2 SW-TLs with LC loading and design equations

The circuit schematic and topology of the considered LC-loaded SW-TLs are depicted in Fig. 1, where $k$, $l$ and $Z_0$ are the phase constant, length and characteristic impedance, respectively, of the host line, and the loading reactive elements are designated by $L_s$ and $C_L$. The dispersion relation (neglecting losses) in the regions of propagation and the characteristic (Bloch) impedance of such lines are given by the following equations [2, 50]

$$\cos(jl) = A, \quad \text{(1)}$$

$$Z_b = \frac{-jb}{\sin(\beta l) \equiv \frac{B'}{\sin(\beta l)}}, \quad \text{(2)}$$

with

$$A = \cos(\kappa l) - \left( \frac{L_s}{2Z_0} + \frac{C_LZ_0}{2} \right) \sin(\kappa l) - \frac{L_sC_L}{2} \cos^2(kl/2), \quad \text{(3)}$$

$$B' = L_s\sin(kl) + \left( \frac{Z_0}{4} - \frac{L_s^2\omega^2}{2Z_0} - \frac{L_sC_L\omega^2}{4Z_0} \right) \sin(kl)$$

$$- \frac{C_L\omega^2Z_0^2}{4} \sin^2(kl/2) - \frac{L_s^2\omega^2}{4} \cos^2(kl/2). \quad \text{(4)}$$
Therefore, the ratio between $f_c/f_0$ must satisfy $f_c/f_0 < 3$. This means that either $\beta l = \pi/2$ ($N = 1$), or $\beta l = \pi/4$ ($N = 2$). However, $\beta l = \pi/4$ is preferred for two main reasons: (i) $f_c$ is further away from $f_0$ (consequently preserving the coupler response in the region of interest), and (ii) the stopband is extended up to higher frequencies (thus enhancing the harmonic suppression capability of the coupler). Thus, we can conclude that the coupler will consist of three-quarter wavelength ($\theta = \pi/2$) SW-TL sections (each one with $N = 2$) and one SW-TL section with $\theta = 3\pi/2$ (and $N = 6$).

Besides (1) and (2), an additional design equation is the slow wave ratio

$$\text{swr} = \frac{\nu_p}{\nu_0} = \frac{\theta/\beta}{\theta/\beta} = \frac{kl}{\beta l}, \quad (8)$$

which provides the level of miniaturisation (in length) of the SW-TL. In (8), $\nu_p$ and $\nu_0$ are the phase velocities of the loaded and unloaded lines.

### 3 Rat-race coupler design

The design process of the rat-race coupler consists of the following steps:

(i) In the first step, the topology of the unit cell, i.e. the one of Fig. 1, described to a good approximation in the region of interest by the circuit model, also shown in Fig. 1, is introduced, and the operating frequency, $f_0$, is set to a certain value (dictated by specifications).

(ii) From the value of swr (a design parameter determining the length reduction of the unit cell), the characteristic impedance $Z_0$ (determined by the specific application, and equal to $Z_0 = 70.71 \Omega$ in a rat-race hybrid coupler) and the required electrical length of the unit cell ($\beta l = 45^\circ$ in our case), we calculate the element parameters of the schematic of Fig. 1a, that is, $L_{ls}$, $C_{ls}$, $Z_0$ and $kl$. Concerning $kl$, it is directly determined from (8), provided $\text{swr}$ is set to a certain value and $\beta l$ is an input parameter ($\beta l = 45^\circ$ in our case). For the determination of $Z_0$, $C_{ls}$ and $L_{ls}$, there is some freedom as long as these three parameters are given by (1) and (2). The constraints are related to the fact that extreme values of $C_{ls}$ and $L_{ls}$ must be avoided since these elements must be implemented in planar form, as quasi-lumped elements. Therefore, these elements are determined from an iterative process, where $Z_{ls}$ is first set to a value equal to $Z_0$, and $L_{ls}$ and $C_{ls}$ are univocally obtained from (1) and (2). Then, the layout of these elements is inferred from Keysight Momentum, where the dimensions are optimised in order to obtain the required reactances at the design frequency (manual tuning is considered). Once these layouts are obtained, it is determined whether the lateral dimensions of both the meander and the patch are comparable or are very different. In the second case, $Z_0$ is varied in order to compensate the lateral size difference between the quasi-lumped reactive elements, taking into account that an increase of $Z_0$ increases $C_{ls}$ and decreases $Z_{ls}$, and vice versa.

(iii) In the third step, once $Z_0$, $L_{ls}$ and $C_{ls}$ have been determined, the individual layouts of the meander, patch and line are assembled, and it is verified if the design goals are satisfied (i.e. if $Z_{ls}$ and $\beta l$ inferred from electromagnetic simulation satisfy the requirements), and the complete layout of the unit cell is optimised if necessary. Note that the length reduction factor cannot exactly coincide with the one given by the selected value of swr, since the meander and the patch have finite dimensions.

(iv) In the next step, the complete circuit layout is assembled, and the topology of the patches is somehow modified, if necessary, to accommodate half of the patches in the inner region of the coupler (note that each capacitance $C_{ls}$ is implemented by means of two parallel patches in order to avoid an excessive patch size).

(v) Finally, the electromagnetic simulation of the whole structure is carried out, and it is optimised again if necessary.
The considered substrate is the Rogers RO4003C with dielectric constant $\varepsilon_r = 3.55$, thickness $h = 1.524$ mm and loss tangent $\tan \delta = 0.0022$. Unit cell dimensions are indicated in the caption of Fig. 3.

The operating frequency of the designed coupler has been set to $f_0 = 0.825$ GHz (central frequency of the E-UTRA Band 20 for LTE). For a rat-race coupler with equal power division between the output ports (hybrid coupler), the characteristic impedance of the constitutive lines should be $Z_0 = 70.71$ Ω. By considering a slow wave ratio of swr = 0.25, and taking into account that the electrical length of the unit cell should be set to $\beta l = \pi/4$ (for the reasons explained before), it follows (using (8)) that $kl = \pi/16$ (further

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**Fig. 2** Dependence of $\beta l$ and $Z_0$ with frequency. The considered substrate is the Rogers RO4003C with dielectric constant $\varepsilon_r = 3.55$, thickness $h = 1.524$ mm and loss tangent $\tan \delta = 0.0022$. Unit cell dimensions are indicated in the caption of Fig. 3.

**Fig. 3** Structure of the designed SW-TL coupler (a) Layout and (b) Photograph. The dimensions, in reference to Fig. 1, are: $W = 1.40$ mm, $b_1 = 6.18$ mm, $b_2 = 5.52$ mm, $W_{p1} = 4.34$ mm, $W_{p2} = 1.70$ mm, $S_2 = 0.60$ mm, $h_2 = 4.865$ mm, $W_s = 0.20$ mm, $W_{s1} = 13.70$ mm, $W_{s2} = 8.18$ mm. The access lines have a width of 3.36 mm, corresponding to 50 Ω characteristic impedance. The area of the indicated (dashed-line) circle is 1880 mm$^2$ (whereas the area of the conventional circular coupler is 8955 mm$^2$).

The proposed SW-TL-based coupler is compared to other compact couplers in the region of interest, the lossless electromagnetic simulation of the designed coupler of Fig. 3e (inferred from Keysight Momentum) is compared to the circuit simulation (obtained from the schematic simulator of Keysight ADS) in Fig. 7. Note that for a proper comparison, losses have been excluded in the electromagnetic simulation (in coherence with the schematic model, where losses are not present). In spite of the relative complexity of the considered device (with 12 unit cells), the agreement between the circuit and electromagnetic simulation is good. Consequently, the circuit model and the design procedure of the coupler, detailed in the previous section, are validated. It should be mentioned that the validity of the model is restricted to the frequency region where the meanders and patches are adequately described by an inductance and a capacitance, respectively. For that reason, the comparison is limited to 2.5 GHz in Fig. 7.

**4 Results and comparison to other compact couplers**

The response of the coupler (including the lossy electromagnetic simulation and measurement) is depicted in Fig. 4. The measurements have been carried out by means of the 4-port Agilent PNA N5221A network analyser, and the calibration has been done using an electronic calibration (Ecal Module N7554A).

The proposed coupler exhibits a high harmonic rejection efficiency, with $>17$ dB suppression up to at least 10 GHz (which means that at least the first five harmonics are strongly suppressed). The phase balance for the $\Sigma$ and $\Lambda$ ports in the vicinity of $f_0$, depicted in Fig. 5, also validates the coupler functionality in the region of interest.

The simulated response of the ordinary coupler, designed at the same frequency and considering identical substrate, is compared to the simulated response of the designed SW-TL-based coupler in Fig. 6. It can be appreciated that the response of the SW-TL-based coupler is roughly indistinguishable from the one of the ordinary coupler in the region of interest (i.e. around $f_0$), which means that the functionality of the coupler in such region is compatible with the elimination of harmonic bands and with a substantial size reduction.

To demonstrate the validity of the circuit schematic of Fig. 1a in the region of interest, the lossless electromagnetic simulation of the designed coupler of Fig. 3e (inferred from Keysight Momentum) is compared to the circuit simulation (obtained from the schematic simulator of Keysight ADS) in Fig. 7. Note that for a proper comparison, losses have been excluded in the electromagnetic simulation (in coherence with the schematic model, where losses are not present). In spite of the relative complexity of the considered device (with 12 unit cells), the agreement between the circuit and electromagnetic simulation is good. Consequently, the circuit model and the design procedure of the coupler, detailed in the previous section, are validated. It should be mentioned that the validity of the model is restricted to the frequency region where the meanders and patches are adequately described by an inductance and a capacitance, respectively. For that reason, the comparison is limited to 2.5 GHz in Fig. 7.

The proposed SW-TL-based coupler is compared to other compact couplers in Tables 1 and 2 (the relative size is in comparison to the size of the ordinary coupler). It should be clarified that two main types of topologies have been considered in the miniaturised couplers of the table: circular (or quasi-circular) and rectangular (or quasi-rectangular). Thus, the comparison with the ordinary counterpart has been done by considering the
equivalent topology. Note that by quasi-circular (or quasi-rectangular) we refer to a rat-race topology similar to the circular (or rectangular) one (e.g. the hexagonal-shaped rat race of Fig. 3 is considered as quasi-circular in this work). For quasi-circular, or quasi-rectangular, miniaturised rat-race couplers, the comparison with the ordinary counterpart has been made by considering the smallest circle, or rectangle, containing the miniaturised topology. The couplers of [41, 42, 48] are extremely small, but their harmonic suppression capability is very limited or null. The coupler presented in [43] exhibits a very good combination of size reduction and harmonic suppression, with only 15.6% the size of the ordinary counterpart, and suppression up to the sixth harmonic. In the coupler presented in this work, the size reduction is also good (21%), and up to the fifth harmonic is suppressed. However, it is remarkable that in the present work the overall harmonic suppression level of the coupler is better than the one found in [43]. Moreover, the bandwidth for matching, power splitting and phase balance is somehow superior in the proposed rat-race. Therefore, the proposed SW-TL-based coupler, implemented by artificial lines with simultaneous LC loading, is found to be competitive in terms of combination of size and harmonic suppression. Moreover, it has been clearly pointed out that the electrical length of the unit cell of the considered periodic lines should not be arbitrary. Particularly, it has been found that unit cells with 45° electrical length are required to maintain the coupler response in the region of interest unaltered and maximise the stopband for harmonic suppression.

Concerning the useful operating range of the coupler, this is given by the bandwidth. Also in Table 2, we have included the bandwidth for $S_{11}$, defined (in this work) as the frequency band where matching is better than $-15$ dB, the bandwidth for power splitting, where the response deviates no $>0.25$ dB from the value at $f_0$ and the bandwidth for phase balance (in this case with a tolerance of $\pm 5^\circ$). The previous bandwidths are compared to those of the other couplers, and we can conclude that the proposed coupler is also competitive in this regard.

Let us now justify the reason for using periodic structures, rather than a filtering non-periodic structure, or a structure with
some weighting function, for the purpose of spurious suppression and size reduction. Periodic structures provide stopbands due to the well-known Bragg effect. However, as compared to filters (where periodicity is sacrificed), periodic structures have intrinsic limitations such as moderate stopband bandwidth and the presence of ripple in the passbands. Several works have been published (e.g., [51]), where, by sacrificing periodicity (through tapering and by continuously varying the unit cell dimensions), ripple is minimised, and the stopband bandwidth is enhanced.

In the present work, the intention is to achieve harmonic suppression and simultaneously reduce the size of the coupler by means of the slow wave effect related to the presence of inductances and capacitances, which effectively reduce the phase velocity of the constitutive artificial lines. Since we do not need, strictly speaking, a stopband filter, but a structure integrated within the coupler able to suppress the harmonic bands, the possible effects of ripple are not relevant in this case. For this main reason, tapering (or weighting) is not necessary in our structure.
other hand, harmonic suppression of (at least) the first five bands is achieved, which is considered to be enough from a practical viewpoint. For all these reasons, the consideration of a purely periodic reactively-loaded artificial line makes sense in this work. Moreover, the structure must behave as a transmission line with an effective phase velocity and characteristic impedance, and its design on the basis of Floquet analysis, as it is done in the paper (mainly through (1), (2) and (8)), is simple and effective. All these important considerations are not so simple by weighting.

To end this section, let us highlight that few simulations have been enough in the optimisation processes indicated in Section 3, carried out manually through parameter sweeping. This concerns element optimisation (individual patch capacitors and meandered inductors), cell optimisation and rat-race optimisation. That is, once the dimensions of the meander inductance and patch capacitance that provide the required reactance at the design frequency are achieved, it is not necessary to alter too much the dimensions in the complete cell in order to optimise the response of such cell. This applies to the overall coupler, as well. Note that optimisation of the meander inductor involves only one variable (its width), whereas the area of the patch capacitors determines the capacitance value. This optimisation has been carried out in order to achieve device functionality at the design frequency. The filtering capability (harmonic suppression) is dictated by the number of cells, \( N \), of each transmission line section, as discussed before. It has been pointed out that \( N \) determines the position of the cutoff frequency (onset of the stop band), and it has been demonstrated that \( N = 2 \) is a good choice, since the region of interest is kept unaltered, whilst the first and subsequent harmonics are suppressed. However, control of the stopband bandwidth cannot be carried out with the proposed approach. We obtain a wide stopband, i.e. an efficient suppression of harmonics (up to the fifth harmonic), due to the fact that many cells are present in the whole device, but the upper cutoff frequency of the stopband is not a design parameter in our approach. Such frequency cannot be predicted with the analytical expressions of Section 2 since the model of Fig. 1 is not valid at high frequencies. Nevertheless, the designed device exhibits very good harmonic suppression capability, this being a strong point of the approach.

5 Conclusions

A compact rat-race coupler with harmonic suppression capability, based on SW-TLs with LC loading, has been reported. A key design aspect to leave unaltered the device response in the region of interest and, simultaneously, efficiently reject the harmonic bands is the number of considered cells (\( N \)), which must be set to \( N = 2 \). In this paper, a systematic design procedure (similar to a design flowchart) of the device, useful for the designers, has been reported as well. As compared to the ordinary coupler, the size has been 79% reduced, and the first five harmonic bands have been 79% suppressed, and the first five harmonic bands have been suppressed with \( >17 \) dB rejection. In certain applications, combining the coupler functionality with low-pass filtering is needed. Thus, space is saved if both functionalities are achieved with a single device, as demonstrated in this work. Moreover, the impact on miniaturisation is further enhanced by means of the proposed SW-TLs. This combination of size and harmonic suppression is competitive. Note that a figure of merit in devices with harmonic/spurious suppression is the number of suppressed spurious bands and the rejection level. With the reported coupler, potential interfering signals at the output ports of the coupler (e.g.

Table 1

<table>
<thead>
<tr>
<th>Ref.</th>
<th>Miniaturisation method</th>
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<tr>
<td>[9]</td>
<td>artificial transmission lines</td>
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<tr>
<td>[29]</td>
<td>shunt open stubs</td>
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<tr>
<td>[40]</td>
<td>capacitor loading</td>
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<td>[41]</td>
<td>slow-wave effect</td>
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<td>[42]</td>
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<tr>
<td>[43]</td>
<td>slow-wave resonant structures</td>
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<tr>
<td>[44]</td>
<td>low impedance and fractal-shaped resonant cells</td>
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<tr>
<td>[45]</td>
<td>periodic stepped-impedance</td>
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<tr>
<td>[46]</td>
<td>T-shaped PBG cells</td>
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<tr>
<td>[47]</td>
<td>capacitive-inductive loading</td>
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<tr>
<td>[48]</td>
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Table 2

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<th>Power splitting bandwidth, % (( \Delta ) ±0.25 dB)</th>
<th>Phase balance bandwidth, % (( \Sigma/\Delta ) ±5( )°)</th>
<th>Area ((l_g)^b)</th>
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\(^a\)The worst rejection level from output ports has been chosen.

\(^b\)\(l_g\) is the guided wavelength.
References


